K22P 1412

Reg. No. : ....

Name : ....

III Semester M.Sc. Degree (CBSS – Reg./Sup./Imp.) Examination, October 2022
(2019 Admission Onwards)
MATHEMATICS
MAT 3 E01 : Graph Theory

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any four** questions from this Part. **Each** question carries **4** marks. **(4×4=16)**

1. Explain the personal assignment problem.
2. Prove that $\alpha+\beta = \nu$.
3. If $\delta > 0$, then prove that $\alpha' + \beta' = \nu$.
4. Show that the Petersen graph is 4-edge-chromatic.
5. Show that $K_5 - e$ is planar for any edge e of K_5.
6. Let u and v be two distinct vertices of the graph G. Then prove that a set S of vertices of G is u-v separating if and only if every u-v path has at least one internal vertex belonging to S.

PART – B

Answer **any four** questions from this Part without omitting **any** Unit. **Each** question carries **16** marks. **(4×16=64)**

UNIT – I

7. a) Prove that if a simple graph G contains no K_{m+1}, then G is degree majorised by some complete m-partite graph H. Also prove that, if G has the same degree sequence as H, then $G \approx H$.

   b) Show that a connected α-critical graph has no cut vertices.

P.T.O.

8. a) For any graph G, prove that $\chi \leq \Delta + 1$.

   b) If G is a connected simple graph and is neither an odd cycle nor a complete graph, then prove that $\chi \leq \Delta$.

9. a) If G is simple, then prove that $\pi_k(G) = \pi_k(G - e) - \pi_k(G.e)$ for any edge e

   b) State and prove Dirac theorem on k-critical graphs.

UNIT – II

10. If G is simple, then prove that either $\chi' = \Delta$ or $\chi' = \Delta + 1$.

11. a) Prove that, inner (outer) bridges avoid one another.

    b) Prove that an inner bridge that avoids every outer bridge is transferable.

12. Prove that the following three statements are equivalent :

    a) every planar graph is 4-vertex-colourable;

    b) every plane graph is 4-face-colourable;

    c) every simple 2-edge-connected 3-regular planar graph is 3-edge-colourable

UNIT – III

13. State and prove Menger's theorem.

14. a) Let G be a bipartite graph with bipartition (X, Y). Then prove that G contains a matching that saturates every vertex in X if and only if $|N(S)| \geq |S|$ for all S

    b) If G is a k-regular bipartite graph with $k > 0$, then G has a perfect matching

15. a) Prove that every 3-regular graph without out edges has a perfect matching

    b) Let l be a feasible vertex labelling of G. If G_l contains a perfect matching M* then prove that M* is an optimal matching of G.

____