

K22P 1412

Reg. No. :

Name :

III Semester M.Sc. Degree (CBSS – Reg./Sup./Imp.) Examination, October 2022
(2019 Admission Onwards)
MATHEMATICS
MAT 3 E01 : Graph Theory

Time : 3 Hours

Max. Marks : 80

PART – A

Answer **any four** questions from this Part. **Each** question carries **4** marks. **(4×4=16)**

1. Explain the personal assignment problem.
2. Prove that $\alpha + \beta = v$.
3. If $\delta > 0$, then prove that $\alpha' + \beta' = v$.
4. Show that the Petersen graph is 4-edge-chromatic.
5. Show that $K_5 - e$ is planar for any edge e of K_5 .
6. Let u and v be two distinct vertices of the graph G . Then prove that a set S of vertices of G is u - v separating if and only if every u - v path has at least one internal vertex belonging to S .

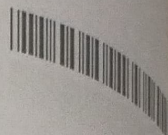
PART – B

Answer **any four** questions from this Part without omitting **any** Unit. **Each** question carries **16** marks. **(4×16=64)**

UNIT – I

7. a) Prove that if a simple graph G contains no K_{m+1} , then G is degree majorised by some complete m -partite graph H . Also prove that, if G has the same degree sequence as H , then $G \approx H$.
b) Show that a connected α -critical graph has no cut vertices.

P.T.O.



8. a) For any graph G , prove that $\chi \leq \Delta + 1$.
b) If G is a connected simple graph and is neither an odd cycle nor a complete graph, then prove that $\chi \leq \Delta$.
9. a) If G is simple, then prove that $\pi_k(G) = \pi_k(G - e) - \pi_k(G, e)$ for any edge e .
b) State and prove Dirac theorem on k -critical graphs.

UNIT – II

10. If G is simple, then prove that either $\chi' = \Delta$ or $\chi' = \Delta + 1$.
11. a) Prove that, inner (outer) bridges avoid one another.
b) Prove that an inner bridge that avoids every outer bridge is transferable.
12. Prove that the following three statements are equivalent :
a) every planar graph is 4-vertex-colourable;
b) every plane graph is 4-face-colourable;
c) every simple 2-edge-connected 3-regular planar graph is 3-edge-colourable.

UNIT – III

13. State and prove Menger's theorem.
14. a) Let G be a bipartite graph with bipartition (X, Y) . Then prove that G contains a matching that saturates every vertex in X if and only if $|N(S)| \geq |S|$ for all $S \subseteq X$.
b) If G is a k -regular bipartite graph with $k > 0$, then G has a perfect matching.
15. a) Prove that every 3-regular graph without cut edges has a perfect matching.
b) Let l be a feasible vertex labelling of G . If G_l contains a perfect matching M^* then prove that M^* is an optimal matching of G .
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